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Analysis and optimal design for bars length errors of overconstrained deployable mechanisms[1]

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Abstract. For the complexity of over constrained deployable mechanism problems, a method to analysis the bars length errors based on the complementary strain energy and the solution of determination of the equilibrium points of mechanism in terms of the minimum complementary strain energy are presented. In this paper, the equilibrium equations are established and the equation of the complementary strain energy is derived for the changes of the complementary strain energy based on over constrained parallelogram deployable mechanism. Then, the optimization model and the corresponding constraints are established with the bars length errors as the design variables and the sum of the complementary strain energy as the goal function. Finally, the improved genetic algorithm is adopted to solve optimization model. The optimization results show that this method can effectively provide a group of optimum bars length errors which can ensure the deployment of the mechanism smooth and steady.

Key words. Deployment mechanism, bars length errors, complementary strain energy, genetic algorithm.

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1. Introduction

Deployable mechanisms, as a late-model space mechanism starting applied in aerospace field, has been more than 30 years since the seventies. Now, it has covered various fields, from each connection part of the rocket to the satellite's solar panels, communication antenna, detection equipment and deployable truss of the space station, etc., and all of them are associated with the concept of deployment mechanisms [1].

Compared with general inerratic mechanisms, the overconstrained mechanism has larger rigidity, and it is able to bear more loads, so it has been widely used in space deployable mechanisms [2]. Because of over constrains existed, it caused the deployable mechanisms was sensitive to errors (manufacturing error, thermal deformation, initial stress, etc.), and it led to a series of harmful effects on deployment behavior. Therefore, analyzing the sensitivity of over constrains to mechanism dimension error and then controlling the number of dimensional accuracy has important research value. Over constrained parallelogram mechanism, existing a single bar length error, was analyzed in Literature [3], but the study, existed more bars length errors and the optimization design of bar length errors, etc., has not been any related literature reported, thus it caused the blindness of deployable mechanism design and the blindness of bars length errors distribution. Therefore, in this paper, based parallelogram deployable mechanism as an example, we analyzed the relationship between mechanism complementary strain energy and deployment angle deeply, and optimized the bars length errors of mechanism, so it could provide a basis for the design of deployable mechanisms.

2. Overconstrained analysis of deployable mechanisms

As shown in Fig. 1 is an over constrained parallelogram deployable mechanism, and it's a common basic construction unit in space deployable mechanisms. The mechanism exists 4 over constrains, consisting of 3 spatial-plane-over constraints and a theoretical-plane-over constraint [4]–[6]. In these over constrains, the spatial-plane-over constraint is sensitive to form and position errors of kinematic pair, while the theoretical-plane-over constraint is due to the repeated bar structure BE or CF, and it's sensitive to bars length errors. In Fig. 1, the bar BE enhances the stiffness of mechanism and the accuracy of deployment, so it's required by the system to work normally. However, unavoidable manufacturing error and thermal deformation effect in the work will make the bar exist bars length errors. If the bar BE existed the bar length error δ, it would make the parallelogram mechanism into a structure, directly influencing kinematic performance of deployable mechanism deployment process. This paper mainly study the bars length errors of parallelogram deployable mechanism influence on mechanism complementary strain energy, on different deployment position.

3. The relationship of the mechanism complementary strain energy and bars length errors

Taking the over constrained parallelogram deployable mechanism in the Fig. 1 as an example, we analyze the relationship between the bars length errors and the complementary strain energy. First we hypothesize that the bars length errors of bar 1, 2, 3 are l_1, l_2, l_3, respectively, and the bar 4, 5 are ideal bars. When these bars of existed errors are fitted together, the complementary strain energy comes into being at the mechanism. As shown in Fig. 2 is bars deformation. In Fig. 2, δ_1, δ_2, δ_3 are deformation of bar 1, 2, 3, respectively, and L_0 is the ideal length. In the analysis, first we suppose the connecting rod (horizontal bar) is a completely rigid bar, which means $EA = \infty$, $EI = \infty$. Next, we suppose, $EA \neq \infty$, $EI = \infty$ and then we suppose $EA \neq \infty$, $EI \neq \infty$. After that we solve the mechanism complementary strain energy in each case. Last, according to the superposition principle, we add all the mechanism complementary strain energy together [7] and we can get total mechanism complementary strain energy.

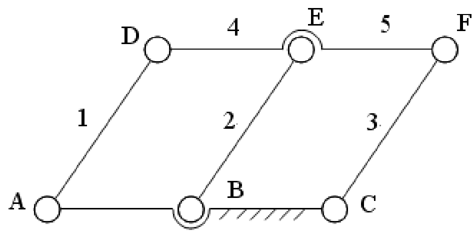

Fig. 1. An over constrained parallelogram deployable mechanism.

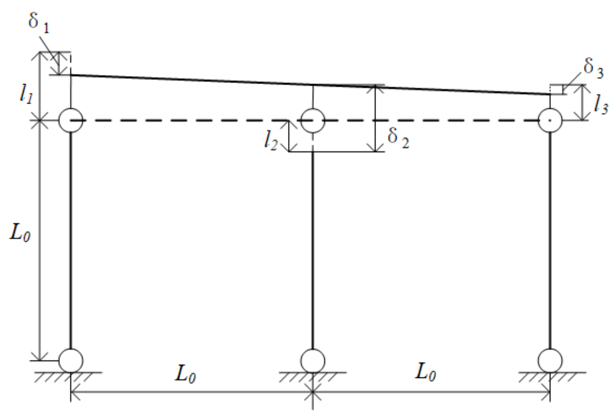

Fig. 2. An over constrained parallelogram mechanism of existed bars length errors.

Mechanism complementary strain energy is calculated using equation

$$U^* = \sum \int \frac{1}{2} \left[\varepsilon_0 P + \gamma_0 Q + k_0 M + \frac{1}{2EA} P^2 + \frac{k}{2GA} Q^2 + \frac{1}{2EI} M^2 \right] \mathrm{d}s \,, \qquad (1)$$

where, U^* is the mechanism complementary strain energy. P, Q, M are section internal force of bars. ε_0, γ_0, k_0 are initial strain. EA is tensile stiffness. k/GA is shear stiffness, and EI is flexural stiffness. Because shear has little influence on the mechanism complementary strain energy, so in the process of calculation, we ignore the effect of shear.

As shown in Fig. 2, when the connecting rod is a completely rigid bar ($EA = \infty$, $EI = \infty$), we can get the following equations

$$\sum Y = N_1 + N_2 + N_3 = 0 \,, \qquad (2)$$

$$\sum M_F = 2N_1 + N_2 = 0 \,, \qquad (3)$$

$$N_1 = N_3 = N \,, \qquad (4)$$

where N_1, N_2, N_3 are section internal force of bars.

The deformations of side link have the following relationship

$$l_1 + \delta_1 + l_3 + \delta_3 = 2(l_2 + \delta_2) \,. \qquad (5)$$

Based on the mechanics of materials, section internal force of bars can be calculated as follows

$$N_1 = \frac{EA\delta_1}{L_0 + l_1}, N_2 = \frac{EA\delta_2}{L_0 + l_2}, N_3 = \frac{EA\delta_3}{L_0 + l_3} \,. \qquad (6)$$

With Eqs. (1)–(5), we can get the following expression

$$\delta_2 = -2 \frac{(2l_2 - l_1 - l_3)\,(L_0 + l_2)}{6L_0 + l_1 + 4l_2 + l_3} \,. \qquad (7)$$

Substituting Eq. (6) into Eq. (5), we can get the following expression of section internal force of bars

$$N_1 = N_3 = -\frac{1}{2} N_2 = \frac{EA\,(2l_2 - l_1 - l_3)}{6L_0 + l_1 + 4l_2 + l_3} = N \,. \qquad (8)$$

When the connecting rod is an axial tension and compression bar ($EA \neq \infty$, $EI = \infty$), the bar deformation of a certain position during the mechanism deployment process of is shown in Fig. 3, in which φ is the opening angle. Here we assume that the axial deformation of the connecting rod is very small so that it only leads to axial force and has no effect on the deformation of the side link. So each side link's deformation still meets the coordination relation.

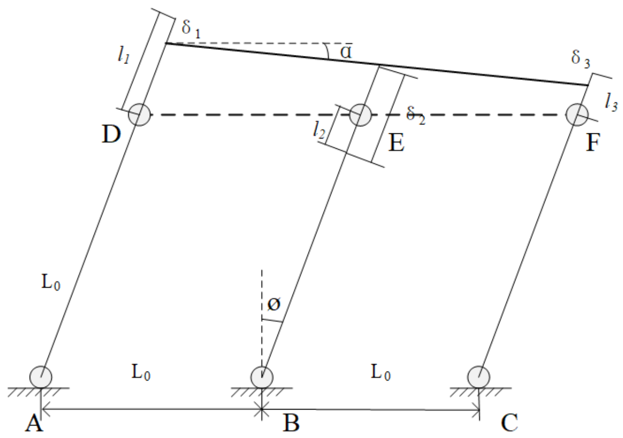

Fig. 3. A certain position during the deployment process of the parallelogram mechanism.

The force on the connecting rod is shown in Fig. 4.

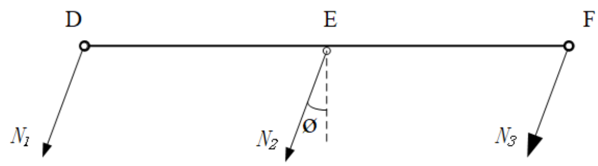

Fig. 4. The forces of the connecting rod (horizontal bar).

According to the equilibrium relation, the following equation can be obtained, where T means connecting rod's axial force

$$T = N \sin\phi\,. \tag{9}$$

When the connecting rod is a beam structure, the connecting rod bending moment is shown in Fig. 5, and it is also assumed that the bending of the connecting rod only affects the internal force of the connecting rod, and has no influence on rod connected to the side link.

The bending moment in point E can be described as follows

$$M = N \cos\phi L_0\,. \tag{10}$$

Substituting Eqs. (8), (9), (10) into Eq. (1), the total complementary strain energy of the mechanism can be obtained

$$U^* = N(l_1+l_3-2l_2)+\frac{N^2}{2EA}(6L_0+4l_2+l_1+l_3+2L_0\sin^2\phi)+\frac{1}{2EI}\frac{2}{3}N^2\cos^2\phi L_0^3\,. \tag{11}$$

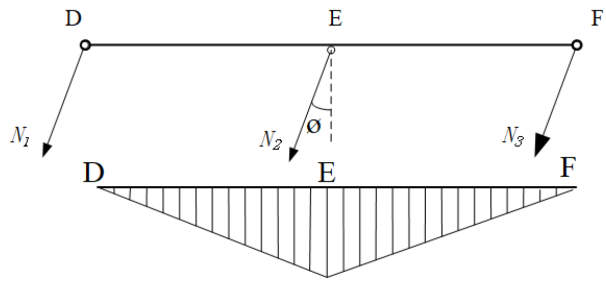

Fig. 5. Connecting rod (horizontal bar) bending moment diagram.

Based on the strain complementary energy principle [8], Eq. (9) can be differentiated as follows

$$\frac{\partial U^*}{\partial N} = (l_1 + l_3 - 2l_2) +$$

$$+\frac{N}{EA}\left(6L_0 + l_1 + 4l_2 + l_3 + 2L_0 \sin^2\phi + \frac{2}{3}\frac{A}{I}\cos^2\phi L_0^3\right) = 0\,. \tag{12}$$

The axial force N can be solved from Eq. (12). Substituting N into Eq. (11), the mechanism complementary strain energy can be obtained as follows

$$U^* = \frac{EA\,(l_1 + l_3 - 2l_2)^2}{2\left(6L_0 + l_1 + 4l_2 + l_3 + 2L_0 \sin^2\phi + \frac{2}{3}\frac{A}{I}\cos^2\phi L_0^3\right)}\,. \tag{13}$$

In this mechanism, if the cross-sectional area of each member bar is $A = 0.15\,\mathrm{m}^2$, modulus of elasticity is $E = 300\,\mathrm{GPa}$ (the material of the deployable mechanism is carbon fiber tube), $A/I = 70$, $L_0 = 2000\,\mathrm{mm}$. Giving bars length errors as follows: $l_1 = 1\,\mathrm{mm}$, $l_2 = -2\,\mathrm{mm}$, $l_3 = 1.5\,\mathrm{mm}$, we can get the mechanism complementary strain energy variation curve as shown in Fig. 6 according to Eq. (13).

The mechanism complementary strain energy reflects the magnitude of the internal energy during the motion of the mechanism. The smaller the strain energy absolute value is, the more stable the mechanism is, and the smoother the mechanism runs. As can be seen in Fig. 6, when the mechanism is in the deployment position, in which $\varphi = 0\,^\circ$, complementary strain energy U^* is the smallest. With the increase of opening angle φ, the value of complementary strain energy U^* increases as well, and when φ increases to about $85\,^\circ$, complementary strain energy reaches the maximum value. This indicates that the stable equilibrium position of the mechanism is its deployment position.

These are the method of analyzing bars length errors and the method of determining the stable equilibrium position, which are based on the complementary strain energy.

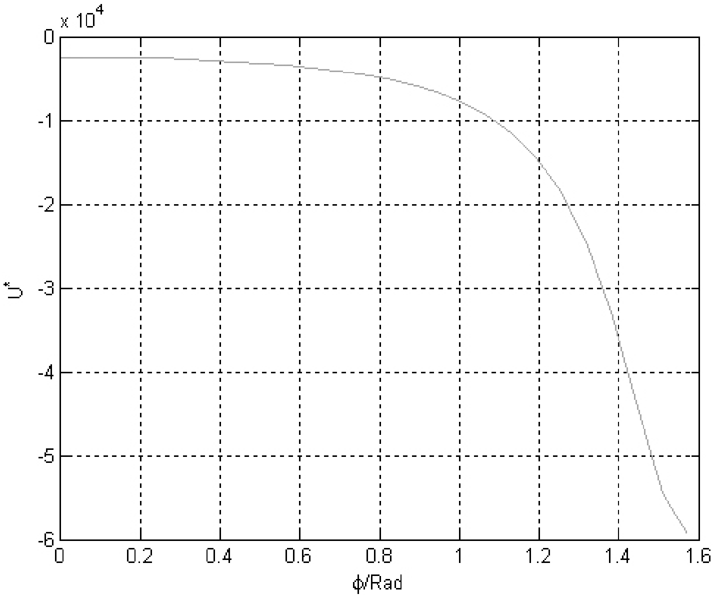

Fig. 6. The mechanism complementary strain energy variation curve.

4. Optimization of bars length errors

From the mechanism complementary strain energy curve, we can see that during the deployment process, the mechanism complementary strain energy changes greatly. The absolute value of the complementary strain energy reflects the fluency of the mechanism's motion. Therefore, it is necessary to optimize the bars length errors, in order to make the mechanism deploy smooth and steady.

4.1. Mathematical model of the optimization

The mathematical model of optimal design is usually considered from three aspects: selecting design variables, listing the objective function and giving constraint condition. Design variables

$$X = (x_1, x_2, x_3) = \left(\frac{l_1}{L_0}, \frac{l_2}{L_0}, \frac{l_3}{L_0} \right) . \tag{14}$$

Objective function

$$\min f\left(X\right) = \sum_{\phi=0}^{\pi/2} \left|U^*\right| =$$

$$= \sum_{\phi=0}^{\pi/2} \frac{EA\left(x_1 + x_3 - 2x_2\right)^2}{2L_0\left(6 + x_1 + 4x_2 + x_3 + 2\sin^2\phi + \frac{2}{3}L_0^2\frac{A}{I}\cos^2\phi\right)}. \tag{15}$$

During the unfolding and folding process, each member bar is subjected to tension compression stress, so the member bar must meet the strength criterion, which requires the tension and compression stress of each member bar stress should not exceed the allowable stress. The bars length errors are also limited by manufacturing process. When the ideal rod length is 6000 mm, constraint conditions are as follows

$$\begin{array}{c} \text{s.t. } \sigma_{\max} = \max\left\{\frac{N_1}{A}, \frac{N_2}{A}, \frac{N_3}{A}\right\} = \frac{2E(x_1+x_3-2x_2)}{6+x_1+4x_2+x_3} =\leq \frac{\sigma_{\lim}}{s}, \\ -0.00005 \leq x_1 \leq -0.00001; \\ 0.00001 \leq x_2 \leq 0.00005; \\ -0.00005 \leq x_3 \leq -0.00001. \end{array} . \tag{16}$$

4.2. Solving the optimization model

Genetic algorithm is a new optimization method which is created by combining the biological evolution principle with optimization design and computer technology. This algorithm uses genetic arithmetic (crossover and mutation) and evolutionary computation (selection) to improve the fitness value of individual in subsequent generations continuously. This paper uses an improved genetic algorithm to solve the above optimization problem [9].

4.3. Numerical examples

As shown in Fig. 3, we choose the model in which horizontal bars' errors are zero to optimize, and the initial design variables are a set of values randomly selected within the range of each bar length error. The optimal solution of the bars length errors is: $X^* = (x_1, x_2, x_3) = (-0.1016, 0.1009, -0.1019)$. The variation curve of the mechanism complementary strain energy in the iterative process is shown in Fig. 7.

As can be seen in Fig. 7, during the deploy process, the total complementary strain energy decreases with the increase of the number of iterations and convergences to a stationary value in the end. With the bars length errors optimized, the total complementary strain energy tends to the minimum value. It indicates that the mechanism's performance has been significantly improved after optimization. The stationary and smoothness of mechanisms get significantly enhanced.

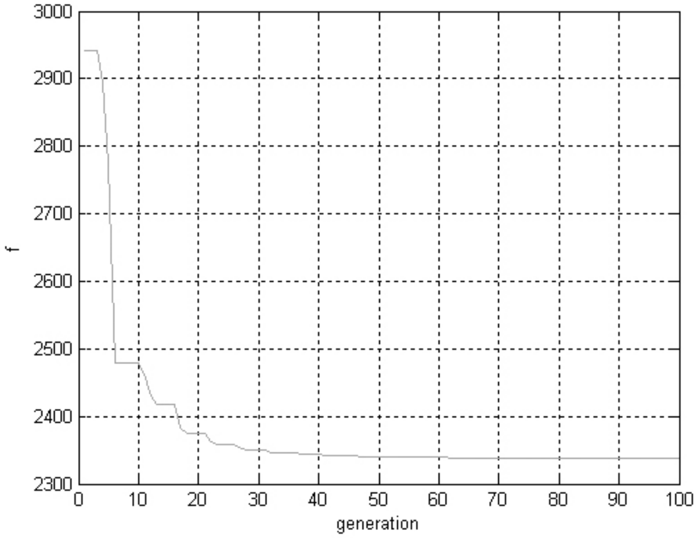

Fig. 7. The variation curve of the objective function value in the iterative process.

5. Conclusions

1. This paper takes a kind of over constrained deployable parallelogram mechanism as example, and thus gets the method of analyzing the bars length errors based on the complementary strain energy. Although the derivation process is carried out on the assumption that the bars length errors of the connecting bars (horizontal bars) are the same value, this analyze method is still applicable for other deployable mechanism or cases in which different bars length error exists.

2. In order to make the mechanism deploy steady and smoothly, the bars length errors of the mechanism is optimized with the improved genetic algorithm and a set of optimal bars length errors is obtained, which can make the sum of mechanism complementary strain energy be the minimum value.

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